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Prediction of the Structural Properties of Washed Gravel Concrete Using Ibearugbulem's Regression Model

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Abstract: **The growing use of washed gravel as a cost-effective and practical alternative to crushed granite in concrete production emphasizes the need to understand its strength properties. This study investigates the development of compressive, flexural, and split tensile strengths in washed gravel concrete at 3, 7, 14, 21, and 28 days, and applies the Ibearugbulem regression model to predict the 28-day strengths. Laboratory strength tests were conducted on 17 concrete mix ratios—12 for model formulation (N1–N12) and 5 control mixes (C1–C5) for validation. The compressive strength test yielded a peak 28-day strength of 30.54 N/mm² (experimental) versus 28.76 N/mm² (predicted) for a mix ratio of 1:1.2:1.5 (w/c = 0.4). Similarly, flexural strength peaked at 7.39 N/mm² (experimental) compared to 6.53 N/mm² (predicted) for a mix ratio of 1:2.5:3 (w/c = 0.55), while the split tensile strength peaked at 2.35 N/mm² (experimental) and 2.14 N/mm² (predicted) for a 1:2:2.5 mix (w/c = 0.6) at 28 days. The accuracy of the model's prediction of the concrete strengths was checked using Fisher's F-test at a 95% confidence level. Our results show that washed gravel is a viable material for concrete production with rapid strength development, with 81% of the 28-day strengths attained in 3 days, and the model reliably predicts these strengths attained.**

Keywords: **Washed gravel concrete, Compressive strength, Flexural strength, Split tensile strength, Regression model, Concrete mix design.**

1. INTRODUCTION

Concrete remains a key component in modern infrastructure development due to its strength, affordability, and ease of application. However, increasing scarcity and the cost of conventional coarse aggregates, such as crushed granite, have necessitated the exploration of locally available alternatives. Washed gravel, a locally sourced material, has demonstrated potential as a viable substitute. This study focuses on modelling and predicting compressive, flexural, and split tensile strengths of concrete made with washed gravel using Ibearugbulem's regression model. By employing pseudo variables derived from actual mix ratios, the model offers a practical solution for predicting strength properties with limited laboratory resources.

Concrete's performance is mainly influenced by the properties of its constituent materials, particularly aggregates, which form 70-80% of its volume [1]. Several studies have explored alternative materials to granite. [2] and [3] demonstrated that washed gravels from local sources can be structurally adequate when properly assessed. Studies by [4], [5], and [6] have revealed that the size, gradation, and cleanliness of washed gravel significantly influence the resulting concrete strength.

Optimization models such as [7] and [8] provide theoretical bases for concrete mix design. The [7] model utilizes polynomial expressions based on component ratios within simplex lattices. Osadebe extended this by applying absolute volumes in an unrestricted design space [9]. These approaches are highly effective for optimizing concrete mixtures. However, they are limited in that they require a predefined number of tests during model development and can only be

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applied to mix ratios within the predetermined experimental range. These constraints restrict their suitability for optimizing already conducted laboratory tests. [10] addressed these limitations with their new regression model by leveraging the algebraic invariance of constants to simplify Osadebe's model. This new model is suitable for concrete mix design using pseudo variables derived from normalized mix ratios that sum up to unity and individually vary from 0 to 1.

Considering an arbitrary amount, S of a given mixture with q components, and the proportion of the i^{th} component of the mixture be S_i. Then from the principle of absolute volume (or mass),

$$\frac{S_1}{S} + \frac{S_2}{S} + \cdots + \frac{S_q}{S} = 1 \tag{1}$$

Where $\frac{S_i}{S}$ is the proportion of the i^{th} constituent of the mixture, $let \quad \frac{S_i}{S} = Z_i$

$$Z_1 + Z_2 + \cdots + Z_q = \sum_{i=1}^{q} Z_i = 1 \tag{2}$$

The response function F(Z), is

$$F(Z) = \sum_{n=0}^{\infty} b_n (Z_i - Z_0)^n \tag{3}$$

Taylor series expansion of equation 3, solving for the coefficients, the resulting equation is summarized as

$$F(Z) = \sum_{n=0}^{\infty} \frac{\partial^n}{\partial Z_i^n} \frac{F(Z_0)}{n!} (Z_i - Z_0)^n \tag{4}$$

By assuming that the response function, F(Z) is continuous and differentiable with respect to its predictors, Z_i, the response can be expanded about

$Z_0 = \left(Z_{0(1)}, Z_{0(2)}, \ldots Z_{0(q)}\right)^T$ as:

$$F(Z) = F(Z_0) + \sum_{i=1}^{q} \frac{\partial F(Z_0)}{\partial Z_i}(Z_i - Z_0) + \sum_{i=1}^{q-1}\sum_{j=1}^{q} \frac{\partial^2}{\partial Z_i \partial Z_j}\frac{F(Z_0)}{2!}\left(Z_i - Z_{0(i)}\right)\left(Z_j - Z_{0(j)}\right) + \sum_{i=1}^{q} \frac{\partial^2}{\partial Z_i^2}\frac{F(Z_0)}{2!}(Z_i - Z_0)^2 + \ldots \tag{5}$$

[10] simplified at $Z_0 = 0$, considering that the product and quotient of constants are also constants to deduce the response function, F(Z) as,

$$F(Z) = b_0 + \sum_{i=1}^{n} b_n {Z_i}^n \tag{6}$$

$$For\ i = q\ , \ 1 \leq n \leq q \tag{7}$$

From equation 2,

$$b_0 = \sum_{i=1}^{q} b_0 Z_i \tag{8}$$

For q = 1

The coefficients are:

$b_n = b_0$ and
$b_n = b_i \; (for\ the\ Z_i\ term)$

For q = 2

The coefficients are:

$$\left.\begin{aligned} b_n &= b_0 \\ b_n &= b_i \; (for\ the\ Z_i\ term) \\ b_n &= b_{ij} \left(for\ the\ Z_i Z_j\ term\right) \\ b_n &= b_{ii} \; (for\ the\ Z_i^2\ term) \end{aligned}\right\} \tag{9}$$

For q = 3

The coefficients are:

$$\left.\begin{aligned} &b_n = b_0\ ; && b_n = b_i \; (for\ the\ Z_i\ term) \\ &b_n = b_{ij} \left(for\ the\ Z_i Z_j\ term\right); && b_n = b_{ii} \; (for\ the\ Z_i^2\ term) \\ &b_n = b_{iii} \; (for\ the\ Z_i^3\ term); && b_n = b_{ijk} \left(for\ the\ Z_i Z_j Z_k\ term\right) \\ &b_n = b_{iij} \left(for\ the\ Z_i^2 Z_j\ term\right); && b_n = b_{ijj} \left(for\ the\ Z_i Z_j^2\ term\right) \\ &b_n = b_{iik} \; (for\ the\ Z_i^2 Z_k\ term) && b_n = b_{ikk} \; (for\ the\ Z_i Z_k^2\ term) \\ &b_n = b_{jjk} \left(for\ the\ Z_j^2 Z_k\ term\right) && b_n = b_{jkk} \left(for\ the\ Z_j Z_k^2\ term\right) \end{aligned}\right\} \tag{10}$$

$$F(Z) = \sum_{i \le q}^{q} \beta_i Z_i + \sum_{1 \le i \le j \le q}^{q} \beta_{ij} Z_i Z_j + \sum_{1 \le i \le j \le k \le q}^{q} \beta_{ijk} Z_i Z_j Z_k + \cdots + \sum_{1 \le i \le j \le k \le \cdots \le q}^{q} \beta_{ijk\ldots q} Z_i Z_j Z_k \ldots Z_q \tag{11}$$

For $i = q = 3$ (3 - component mixture)

$$F(Z) = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{23} Z_2 Z_3 + \beta_{123} Z_1 Z_2 Z_3 \tag{12}$$

Equation 11 can be summed up to form simultaneous equations summarized in equation 13.

$$\begin{bmatrix} \sum_{r \le n}^{n} Z_1 F(Z) \\ \sum_{r \le n}^{n} Z_2 F(Z) \\ \sum_{r \le n}^{n} Z_3 F(Z) \\ \vdots \\ \sum_{r \le n}^{n} Z_1 Z_2 Z_3 \ldots F(Z) \end{bmatrix} = \begin{bmatrix} \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_1 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_2 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_3 & \cdots \\ \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_2 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_2 Z_2 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_2 Z_3 & \cdots \\ \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_3 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_2 Z_3 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_3 Z_3 & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_1 Z_2 Z_3 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_2 Z_2 Z_3 & \sum_{r \le n}^{n} \sum_{i \le q}^{q} Z_1 Z_2 Z_3 Z_3 & \cdots \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{123} \end{bmatrix} \tag{13}$$

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$$\begin{bmatrix} \sum Z_1F(Z) \\ \sum Z_2F(Z) \\ \sum Z_3F(Z) \\ \sum Z_1Z_2F(Z) \\ \sum Z_1Z_3F(Z) \\ \sum Z_2Z_3F(Z) \\ \sum Z_1Z_2Z_3F(Z) \end{bmatrix} = \begin{bmatrix} \sum\sum Z_1Z_1 & \sum\sum Z_1Z_2 & \sum\sum Z_1Z_3 & \sum\sum Z_1Z_1Z_2 & \sum\sum Z_1Z_1Z_3 & \sum\sum Z_1Z_2Z_3 & \sum\sum Z_1Z_1Z_2Z_3 \\ \sum\sum Z_1Z_2 & \sum\sum Z_2Z_2 & \sum\sum Z_2Z_3 & \sum\sum Z_1Z_2Z_2 & \sum\sum Z_1Z_2Z_3 & \sum\sum Z_2Z_2Z_3 & \sum\sum Z_1Z_2Z_2Z_3 \\ \sum\sum Z_1Z_3 & \sum\sum Z_2Z_3 & \sum\sum Z_3Z_3 & \sum\sum Z_1Z_2Z_3 & \sum\sum Z_1Z_3Z_3 & \sum\sum Z_2Z_3Z_3 & \sum\sum Z_1Z_2Z_3Z_3 \\ \sum\sum Z_1Z_1Z_2 & \sum\sum Z_1Z_2Z_2 & \sum\sum Z_1Z_2Z_3 & \sum\sum Z_1Z_1Z_2Z_2 & \sum\sum Z_1Z_1Z_2Z_3 & \sum\sum Z_1Z_2Z_2Z_3 & \sum\sum Z_1Z_1Z_2Z_2Z_3 \\ \sum\sum Z_1Z_1Z_3 & \sum\sum Z_1Z_2Z_3 & \sum\sum Z_1Z_3Z_3 & \sum\sum Z_1Z_1Z_2Z_3 & \sum\sum Z_1Z_1Z_3Z_3 & \sum\sum Z_1Z_2Z_3Z_3 & \sum\sum Z_1Z_1Z_2Z_3Z_3 \\ \sum\sum Z_1Z_2Z_3 & \sum\sum Z_2Z_2Z_3 & \sum\sum Z_2Z_3Z_3 & \sum\sum Z_1Z_2Z_2Z_3 & \sum\sum Z_1Z_2Z_3Z_3 & \sum\sum Z_2Z_2Z_3Z_3 & \sum\sum Z_1Z_2Z_2Z_3Z_3 \\ \sum\sum Z_1Z_1Z_2Z_3 & \sum\sum Z_1Z_2Z_2Z_3 & \sum\sum Z_1Z_2Z_3Z_3 & \sum\sum Z_1Z_1Z_2Z_2Z_3 & \sum\sum Z_1Z_1Z_2Z_3Z_3 & \sum\sum Z_1Z_2Z_2Z_3Z_3 & \sum\sum Z_1Z_1Z_2Z_2Z_3Z_3 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \\ \beta_{123} \end{bmatrix} \quad (14)$$

Simplified as,

$$[F(Z)] = [CC][\beta] \quad (15)$$

2. MATERIALS AND METHODS

In this study, Portland Limestone Cement (PLC), river sand sourced from Calabar metropolis, washed gravel obtained from Akamkpa LGA, Cross River state, Nigeria, and clean potable water were used for concrete production. A total of seventeen concrete mixes were prepared, consisting of twelve development mixes (N1-N12) for model formulation and five control mixes (C1-C5) for validation. The mix designs incorporated varying aggregate proportions and water-cement ratios ranging from 0.4 to 0.65 to evaluate a spectrum of concrete properties.

Comprehensive aggregate characterization was conducted, including sieve analysis performed according to ASTM C136/C33 standards for both fine and coarse aggregates. Additional aggregate tests performed included specific gravity and water absorption (ASTM C128/C127), Aggregate Impact Value (BS 812-112:1990), and bulk density (ASTM C29/C29M). The workability of fresh concrete was evaluated using slump tests following ASTM C143 procedures.

Strength testing was conducted following British Standards, with compressive strength measured at 3, 7, 14, 21, and 28 days (BS 1881-116). Flexural strength was determined through third-point loading of beams in accordance with BS 1881-118, while split tensile strength was evaluated using the indirect cylinder test method specified in BS 1881-117. The experimental results were optimized using the Ibearugbulem regression model, with model reliability verified through Fisher's F-test at a 95% confidence level. This comprehensive testing protocol ensured robust evaluation of washed gravel concrete properties and validation of the predictive model's accuracy across different mix designs for 28-day strengths.

The summary of the strength tests are shown in Table1, Table2, and Table3.

Table 1: Compressive Strengths for the different mix ratios at 3, 7, 14 ,21 and 28 days

Compressive strength test result (N/mm^2)							
S/N	**Mix ratio**	**Water/Cement ratio**	**3days**	**7days**	**14days**	**21days**	**28days**
N_1	1:1.2:1.5	0.45	25.26	26.65	27.33	28.37	29.12
N_2	1:1.2:1.5	0.4	26.10	27.74	28.98	29.06	30.54
N_3	1:1.2:1.5	0.55	24.32	24.50	25.80	26.52	27.28
N_4	1:2:2.5	0.55	24.74	25.53	26.02	27.82	28.52
N_5	1:2:2.5	0.5	24.82	25.08	26.42	28.28	29.13
N_6	1:2:2.5	0.6	18.69	19.19	21.40	25.06	27.12
N_7	1:2.5:3	0.55	19.85	20.74	21.02	22.81	25.57
N_8	1:2.5:3	0.65	17.84	18.07	19.31	21.65	23.95
N_9	1:2:4	0.63	21.88	22.51	23.57	24.48	25.64
N_{10}	1:1.5:3	0.6	20.77	21.71	22.38	24.71	25.81
N_{11}	1:1.5:3	0.55	19.83	20.01	22.82	25.59	27.11
N_{12}	1:2:4	0.55	23.21	23.82	24.85	25.12	26.87
C_1	1:1.5:3	0.5	24.51	25.44	25.98	26.94	27.13
C_2	1:2:4	0.5	24.64	25.27	25.95	26.42	26.91
C_3	1:1.2:1.5	0.5	23.96	24.12	24.93	26.71	27.95
C_4	1:1:2	0.5	22.87	23.48	23.70	24.24	24.96
C_5	1:2:2.5	0.65	24.23	25.62	25.88	26.57	27.54

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Table 2: Flexural strengths for the different mix ratios at 3, 7, 14 ,21 and 28 days

Flexural strength test result (N/mm²)							
S/N	**Mix ratio**	**Water/Cement ratio**	**3days**	**7days**	**14days**	**21days**	**28days**
N_1	1:1.2:1.5	0.45	4.28	4.29	5.03	5.28	5.36
N_2	1:1.2:1.5	0.4	4.52	4.77	5.11	5.94	7.28
N_3	1:1.2:1.5	0.55	4.12	4.28	4.47	4.58	4.61
N_4	1:2:2.5	0.55	5.72	5.76	5.52	5.67	5.86
N_5	1:2:2.5	0.5	4.66	5.27	5.29	5.30	5.33
N_6	1:2:2.5	0.6	5.80	5.96	5.98	6.15	6.20
N_7	1:2.5:3	0.55	6.82	6.92	6.93	7.10	7.39
N_8	1:2.5:3	0.65	4.69	4.71	4.82	4.89	5.82
N_9	1:2:4	0.63	5.38	5.48	5.19	5.23	5.36
N_{10}	1:1.5:3	0.6	4.52	4.65	4.78	4.55	5.85
N_{11}	1:1.5:3	0.55	4.34	4.37	4.60	5.01	6.32
N_{12}	1:2:4	0.55	5.31	5.39	5.67	5.78	5.82
C_1	1:1.5:3	0.5	5.55	5.59	5.67	5.81	6.23
C_2	1:2:4	0.5	5.18	5.22	5.26	5.29	5.36
C_3	1:1.2:1.5	0.5	5.31	5.36	5.37	5.33	5.39
C_4	1:1:2	0.5	5.31	5.06	4.85	4.93	5.05
C_5	1:2:2.5	0.65	5.26	5.29	5.38	5.58	5.98

Table 3: Split tensile strengths for the different mix ratios at 3, 7, 14 ,21 and 28 days

Split tensile strength test result (N/mm²)							
S/N	**Mix ratio**	**Water/Cement ratio**	**3days**	**7days**	**14days**	**21days**	**28days**
N_1	1:1.2:1.5	0.45	1.71	1.75	1.81	1.83	1.88
N_2	1:1.2:1.5	0.4	1.80	1.84	1.85	1.91	2.08
N_3	1:1.2:1.5	0.55	1.25	1.34	1.41	1.52	1.95
N_4	1:2:2.5	0.55	1.85	1.99	1.87	1.90	2.04
N_5	1:2:2.5	0.5	2.16	2.38	2.11	2.01	2.32
N_6	1:2:2.5	0.6	1.94	2.02	2.03	2.08	2.35
N_7	1:2.5:3	0.55	1.25	1.59	1.64	1.71	1.99
N_8	1:2.5:3	0.65	1.17	1.26	1.42	1.50	1.74
N_9	1:2:4	0.63	1.09	1.33	1.33	1.61	1.84
N_{10}	1:1.5:3	0.6	1.00	1.16	1.17	1.24	1.39
N_{11}	1:1.5:3	0.55	1.01	1.20	1.26	1.29	1.52
N_{12}	1:2:4	0.55	1.13	1.38	1.42	1.50	1.68
C_1	1:1.5:3	0.5	1.65	1.76	1.79	1.86	1.90
C_2	1:2:4	0.5	1.11	1.26	1.33	1.44	1.95
C_3	1:1.2:1.5	0.5	1.41	1.58	1.59	1.63	1.88
C_4	1:1:2	0.55	1.25	1.41	1.32	1.37	1.43
C_5	1:2:2.5	0.65	1.39	1.41	1.56	1.74	2.03

We used Ibearugbulem's regression model to generate mixture response function to predict the 28th day strength of concrete. For three component mixtures (S_1, S_2 and S_3) representing water/cement, fine aggregate/cement and coarse aggregate/cement ratios respectively, the pseudo variables are summarized in **Error! Reference source not found.**.

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Table 4: Pseudo variables

S/N	S_1	S_2	S_3	S	Z_1	Z_2	Z_3
N_1	0.45	1.2	1.5	3.15	0.1429	0.3810	0.4762
N_2	0.4	1.2	1.5	3.1	0.1290	0.3871	0.4839
N_3	0.55	1.2	1.5	3.25	0.1692	0.3692	0.4615
N_4	0.55	2	2.5	5.05	0.1089	0.3960	0.4950
N_5	0.5	2	2.5	5	0.1000	0.4000	0.5000
N_6	0.6	2	2.5	5.1	0.1176	0.3922	0.4902
N_7	0.55	2.5	3	6.05	0.0909	0.4132	0.4959
N_8	0.65	2.5	3	6.15	0.1057	0.4065	0.4878
N_9	0.63	2	4	6.63	0.0950	0.3017	0.6033
N_{10}	0.6	1.5	3	5.1	0.1176	0.2941	0.5882
N_{11}	0.55	1.5	3	5.05	0.1089	0.2970	0.5941
N_{12}	0.55	2	4	6.55	0.0840	0.3053	0.6107

Table 5: Pseudo variables continue

S/N	Z_1Z_1	Z_1Z_2	Z_1Z_3	$Z_1Z_1Z_2$	$Z_1Z_1Z_3$	$Z_1Z_2Z_3$	$Z_1Z_1Z_2Z_3$	Z_2Z_2	Z_2Z_3
N_1	0.0204	0.0544	0.0680	0.0078	0.0097	0.0259	0.0037	0.1451	0.1814
N_2	0.0166	0.0499	0.0624	0.0064	0.0081	0.0242	0.0031	0.1498	0.1873
N_3	0.0286	0.0625	0.0781	0.0106	0.0132	0.0288	0.0049	0.1363	0.1704
N_4	0.0119	0.0431	0.0539	0.0047	0.0059	0.0214	0.0023	0.1568	0.1961
N_5	0.0100	0.0400	0.0500	0.0040	0.0050	0.0200	0.0020	0.1600	0.2000
N_6	0.0138	0.0461	0.0577	0.0054	0.0068	0.0226	0.0027	0.1538	0.1922
N_7	0.0083	0.0376	0.0451	0.0034	0.0041	0.0186	0.0017	0.1708	0.2049
N_8	0.0112	0.0430	0.0516	0.0045	0.0054	0.0210	0.0022	0.1652	0.1983
N_9	0.0090	0.0287	0.0573	0.0027	0.0054	0.0173	0.0016	0.0910	0.1820
N_{10}	0.0138	0.0346	0.0692	0.0041	0.0081	0.0204	0.0024	0.0865	0.1730
N_{11}	0.0119	0.0323	0.0647	0.0035	0.0070	0.0192	0.0021	0.0882	0.1765
N_{12}	0.0071	0.0256	0.0513	0.0022	0.0043	0.0157	0.0013	0.0932	0.1865
Σ	**0.1626**	**0.4979**	**0.7093**	**0.0593**	**0.0831**	**0.2550**	**0.0300**	**1.5969**	**2.2485**

Table 6: Pseudo variable continue

S/N	Z_3Z_3	$Z_1Z_2Z_2$	$Z_2Z_2Z_3$	$Z_1Z_2Z_3Z_2$	$Z1Z_3Z_3$	$Z_2Z_3Z_3$	$Z_1Z_2Z_3Z_3$	$Z_1Z_1Z_2Z_3Z_2$
N_1	0.2268	0.0207	0.0691	0.0099	0.0324	0.0864	0.0123	0.0014
N_2	0.2341	0.0193	0.0725	0.0094	0.0302	0.0906	0.0117	0.0012
N_3	0.2130	0.0231	0.0629	0.0106	0.0360	0.0787	0.0133	0.0018
N_4	0.2451	0.0171	0.0776	0.0085	0.0267	0.0971	0.0106	0.0009
N_5	0.2500	0.0160	0.0800	0.0080	0.0250	0.1000	0.0100	0.0008
N_6	0.2403	0.0181	0.0754	0.0089	0.0283	0.0942	0.0111	0.0010
N_7	0.2459	0.0155	0.0847	0.0077	0.0224	0.1016	0.0092	0.0007
N_8	0.2380	0.0175	0.0806	0.0085	0.0251	0.0967	0.0102	0.0009
N_9	0.3640	0.0086	0.0549	0.0052	0.0346	0.1098	0.0104	0.0005
N_{10}	0.3460	0.0102	0.0509	0.0060	0.0407	0.1018	0.0120	0.0007
N_{11}	0.3529	0.0096	0.0524	0.0057	0.0384	0.1048	0.0114	0.0006
N_{12}	0.3729	0.0078	0.0569	0.0048	0.0313	0.1139	0.0096	0.0004
Σ	**3.3290**	**0.1836**	**0.8180**	**0.0931**	**0.3712**	**1.1756**	**0.1318**	**0.0110**

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Table 7: Pseudo variables continue

S/N	$Z_1Z_1Z_2Z_3Z_3$	$Z_2Z_2Z_3Z_3$	$Z_1Z_2Z_2Z_3Z_3$	$Z_1Z_1Z_2Z_2Z_3Z_3$	$Z_1Z_1Z_2Z_2$	$Z_1Z_1Z_3Z_3$
N_1	0.0018	0.0329	0.0047	0.0007	0.0030	0.0046
N_2	0.0015	0.0351	0.0045	0.0006	0.0025	0.0039
N_3	0.0023	0.0290	0.0049	0.0008	0.0039	0.0061
N_4	0.0012	0.0384	0.0042	0.0005	0.0019	0.0029
N_5	0.0010	0.0400	0.0040	0.0004	0.0016	0.0025
N_6	0.0013	0.0370	0.0043	0.0005	0.0021	0.0033
N_7	0.0008	0.0420	0.0038	0.0003	0.0014	0.0020
N_8	0.0011	0.0393	0.0042	0.0004	0.0018	0.0027
N_9	0.0010	0.0331	0.0031	0.0003	0.0008	0.0033
N_{10}	0.0014	0.0299	0.0035	0.0004	0.0012	0.0048
N_{11}	0.0012	0.0311	0.0034	0.0004	0.0010	0.0042
N_{12}	0.0008	0.0348	0.0029	0.0002	0.0007	0.0026
Σ	**0.0153**	**0.4227**	**0.0476**	**0.0056**	**0.0219**	**0.0429**

From equation 14, the elements of the CC-matrix are shown in Error! Reference source not found. and the inverse in Table 9**.**

Table 8: Elements of Matrix [CC]

CC-Matrix						
0.1626	0.4979	0.7093	0.0593	0.0831	0.2550	0.0300
0.4979	1.5969	2.2485	0.1836	0.2550	0.8180	0.0931
0.7093	2.2485	3.3290	0.2550	0.3712	1.1756	0.1318
0.0593	0.1836	0.2550	0.0219	0.0300	0.0931	0.0110
0.0831	0.2550	0.3712	0.0300	0.0429	0.1318	0.0153
0.2550	0.8180	1.1756	0.0931	0.1318	0.4227	0.0476
0.0300	0.0931	0.1318	0.0110	0.0153	0.0476	0.0056

Table 9: Elements of the Inverse of Matrix $[CC]^{-1}$

CC-Matrix Inverse						
14189776.88	3110697.61	1376843.95	-46476394.80	-29371721.28	-8881851.81	87612896.92
3169654.70	912367.12	405881.99	-10781122.36	-6814893.14	-2614875.19	20490893.11
1407083.01	406790.39	182199.64	-4794532.48	-3041093.10	-1169567.98	9154385.98
-46703540.68	-10637812.79	-4717529.28	154088786.05	97344184.56	30426235.80	-291550958.67
-29522776.42	-6725971.07	-2993157.60	97368767.96	61621208.26	19264247.45	-184445110.53
-9068886.52	-2618973.94	-1168788.46	30896660.60	19556844.16	7518213.39	-58880645.30
88376104.76	20294181.60	9041733.48	-292648244.44	-185092220.73	-58203541.36	556168972.10

The F(Z).Z matrices obtained from the laboratory tests are shown in Table 10, Table 11 and Table 12 for compressive, flexural and split tensile strengths respectively.

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Table 10: F(Z). Z (Compressive strength)

S/N	Strength (Lab) = F(z) (N/mm²)	∑Z1.F(z)	∑Z2.F(Z)	∑Z3.F(z)	∑Z1Z2.F(z)	∑Z1Z3.F(z)	∑Z2Z3.F(z)	∑Z1Z2Z3.F(z)
N_1	29.12	4.16	11.0933	13.8667	1.5848	1.981	5.2825	0.7546
N_2	30.54	3.9406	11.8219	14.7774	1.5254	1.9068	5.7203	0.7381
N_3	27.28	4.6166	10.0726	12.5908	1.7046	2.1307	4.6489	0.7867
N_4	28.52	3.1061	11.295	14.1188	1.2302	1.5377	5.5916	0.609
N_5	29.13	2.913	11.652	14.565	1.1652	1.4565	5.826	0.5826
N_6	27.12	3.1906	10.6353	13.2941	1.2512	1.564	5.2134	0.6133
N_7	25.57	2.3245	10.5661	12.6793	0.9606	1.1527	5.2394	0.4763
N_8	23.95	2.5313	9.7358	11.6829	1.029	1.2348	4.7492	0.5019
N_9	25.64	2.4364	7.7345	15.4691	0.735	1.4699	4.6664	0.4434
N_{10}	25.81	3.0365	7.5912	15.1824	0.8931	1.7862	4.4654	0.5253
N_{11}	27.11	2.9526	8.0525	16.105	0.877	1.754	4.7836	0.521
N_{12}	26.87	2.2563	8.2046	16.4092	0.6889	1.3779	5.0104	0.4207
	∑	**37.4645**	**118.4549**	**170.7406**	**13.6448**	**19.3521**	**61.1971**	**6.9731**

Table 11: F(Z). Z (Flexural strength)

S/N	Strength (Lab) = F(z) (N/mm²)	∑Z1.F(z)	∑Z2.F(z)	∑Z3.F(z)	∑Z1Z2.F(z)	∑Z1Z3.F(z)	∑Z2Z3.F(z)	∑Z1Z2Z3.F(z)
N_1	5.36	0.7657	2.0419	2.5524	0.2917	0.3646	0.9723	0.7546
N_2	7.28	0.9394	2.8181	3.5226	0.3636	0.4545	1.3636	0.7381
N_3	4.61	0.7802	1.7022	2.1277	0.2881	0.3601	0.7856	0.7867
N_4	5.86	0.6382	2.3208	2.901	0.2528	0.3159	1.1489	0.609
N_5	5.33	0.533	2.132	2.665	0.2132	0.2665	1.066	0.5826
N_6	6.2	0.7294	2.4314	3.0392	0.286	0.3576	1.1918	0.6133
N_7	7.39	0.6718	3.0537	3.6645	0.2776	0.3331	1.5142	0.4763
N_8	5.82	0.6151	2.3659	2.839	0.25	0.3001	1.1541	0.5019
N_9	5.36	0.5093	1.6169	3.2338	0.1536	0.3073	0.9755	0.4434
N_{10}	5.85	0.6882	1.7206	3.4412	0.2024	0.4048	1.0121	0.5253
N_{11}	6.32	0.6883	1.8772	3.7545	0.2045	0.4089	1.1152	0.521
N_{12}	5.82	0.4887	1.7771	3.5542	0.1492	0.2984	1.0853	0.4207
	∑	**8.0474**	**25.8577**	**37.295**	**2.9328**	**4.1719**	**13.3846**	**6.9731**

Table 12: F(Z). Z (Split tensile strength)

S/N	Strength (Lab) = F(z) (N/mm²)	∑Z1.F(z)	∑Z2.F(z)	∑Z3.F(z)	∑Z1Z2.F(z)	∑Z1Z3.F(z)	∑Z2Z3.F(z)	∑Z1Z2Z3.F(z)
N_1	1.88	0.2686	0.7162	0.8952	0.1023	0.1279	0.341	0.0487
N_2	2.08	0.2684	0.8052	1.0065	0.1039	0.1299	0.3896	0.0503
N_3	1.95	0.33	0.72	0.9	0.1218	0.1523	0.3323	0.0562
N_4	2.04	0.2222	0.8079	1.0099	0.088	0.11	0.4	0.0436
N_5	2.32	0.232	0.928	1.16	0.0928	0.116	0.464	0.0464
N_6	2.35	0.2765	0.9216	1.152	0.1084	0.1355	0.4517	0.0531
N_7	1.99	0.1809	0.8223	0.9868	0.0748	0.0897	0.4078	0.0371
N_8	1.74	0.1839	0.7073	0.8488	0.0748	0.0897	0.345	0.0365
N_9	1.84	0.1748	0.5551	1.1101	0.0527	0.1055	0.3349	0.0318
N_{10}	1.39	0.1635	0.4088	0.8176	0.0481	0.0962	0.2405	0.0283
N_{11}	1.52	0.1655	0.4515	0.903	0.0492	0.0983	0.2682	0.0292
N_{12}	1.68	0.1411	0.513	1.026	0.0431	0.0861	0.3133	0.0263
	∑	**2.6074**	**8.3568**	**11.8158**	**0.9599**	**1.3372**	**4.2883**	**0.4875**

Substituting Table 9, and consecutively Table 10, Table 11, and Table 12 into equation 14, the coefficients β for the response function in equation 12 for compressive, flexural and split tensile strengths are shown in Table 13.

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Table 13: Coefficient of the modeled response function

Coefficients of the response function, β.			
	Compressive Strength	**Flexural Strength**	**Split Tensile Strength**
β_1	-453.103	175.169	118.576
β_2	-1668.577	370.753	-175.197
β_3	-664.103	153.648	-71.516
β_{12}	2896.889	-1548.542	-88.267
β_{13}	1450.120	-755.444	-131.266
β_{23}	4610.403	-1021.316	492.327
β_{123}	-1833.842	3366.324	395.942

The modeled response function for the 28-day strengths of the washed-gravel concrete are shown in equation 16, 17 and 18 for compressive, flexural and split tensile strengths respectively.

Response Function for the Compressive Strength

$$F(Z) = -453.103\,Z_1 - 1668.577\,Z_2 - 664.103\,Z_3 + 2896.889\,Z_1Z_2 + 1450.12\,Z_1Z_3 + 4610.403\,Z_2Z_3 - 1833.842\,Z_1Z_2Z_3 \qquad (16)$$

Response Function for the Flexural Strength

$$F(Z) = 175.169\,Z_1 + 370.753\,Z_2 + 153.648\,Z_3 - 1548.542\,Z_1Z_2 - 755.444\,Z_1Z_3 - 1021.316\,Z_2Z_3 + 3366.324\,Z_1Z_2Z_3 \qquad (17)$$

Response Function for the Split Tensile Strength

$$F(Z) = 118.576\,Z_1 - 175.197\,Z_2 - 71.516\,Z_3 - 88.267\,Z_1Z_2 - 131.266\,Z_1Z_3 + 492.327\,Z_2Z_3 + 395.942\,Z_1Z_2Z_3 \qquad (18)$$

Table 14: Concrete strength prediction for the mix ratios N_1 to N_{12} using the model

Concrete Strengths						
	Compressive Strength (N/mm^2)		Flexural Strength (N/mm^2)		Split Tensile Strength (N/mm^2)	
S/N	Experiment	Model	Experiment	Model	Experiment	Model
N_1	29.12	28.51	5.36	5.73	1.88	1.98
N_2	30.54	28.76	7.28	6.01	2.08	2.06
N_3	27.28	27.79	4.61	4.72	1.95	1.92
N_4	28.52	28.96	5.86	6.09	2.04	2.22
N_5	29.13	28.99	5.33	5.99	2.32	2.31
N_6	27.12	28.89	6.2	6.11	2.35	2.14
N_7	25.57	24.73	7.39	6.53	1.99	1.94
N_8	23.95	24.88	5.82	6.73	1.74	1.79
N_9	25.64	26.47	5.36	5.83	1.84	1.66
N_{10}	25.81	26.20	5.85	5.99	1.39	1.45
N_{11}	27.11	26.33	6.32	5.98	1.52	1.52
N_{12}	26.87	26.53	5.82	5.57	1.68	1.80

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Table 15: Concrete strength prediction for the control mix ratios C_1 to C_5 using the model

Concrete Strengths						
	Compressive Strength (N/mm^2)		Flexural Strength (N/mm^2)		Split Tensile Strength (N/mm^2)	
S/N	Experiment	Model	Experiment	Model	Experiment	Model
C1	27.13	26.43	6.23	5.91	1.90	1.61
C2	26.91	26.53	5.36	5.34	1.95	1.89
C3	27.95	28.19	5.39	5.29	1.88	1.94
C4	24.96	25.61	5.05	5.65	1.43	1.31
C5	27.54	28.79	5.98	6.05	2.03	2.08

To assess the accuracy of the model predictions, Fisher's F-test was performed on the control mix (C1–C5) to determine whether there was a statistically significant difference between the laboratory-measured concrete strengths and the model-predicted strengths at a 95% confidence level.

Table 16: Fisher's F-Test for Compressive Strength

S/N	F_1	F_2	$F_1 - \bar{F_1}$	$F_1 - \bar{F_2}$	$(F_1 - \bar{F_1})^2$	$(F_1 - \bar{F_2})^2$
C1	27.13	26.43	0.23	-0.68	0.054	0.464
C2	26.91	26.53	0.01	-0.58	0.000	0.335
C3	27.95	28.19	1.05	1.08	1.107	1.164
C4	24.96	25.61	-1.94	-1.50	3.756	2.253
C5	27.54	28.79	0.64	1.68	0.412	2.828
				$\sum$	5.329	7.044
Number of Observation, n	5	5				
Degree of freedom DF, (n-1)	4	4				
Mean, $\bar{F}$	26.90	27.11				
Variance, σ^2	1.332	1.761				
F-statistics	$\frac{1.761}{1.332} = 1.322$					
F-critical for $\alpha = 0.05$	6.388					

Table 17: Fisher's F-Test for Flexural Strength

S/N	F_1	F_2	$F_1 - \bar{F_1}$	$F_1 - \bar{F_2}$	$(F_1 - \bar{F_1})^2$	$(F_1 - \bar{F_2})^2$
C1	6.23	5.91	0.23	-0.68	0.054	0.464
C2	5.36	5.34	0.01	-0.58	0.000	0.335
C3	5.394	5.29	1.05	1.08	1.107	1.164
C4	5.05	5.65	-1.94	-1.50	3.756	2.253
C5	5.98	6.05	0.64	1.68	0.412	2.828
				$\sum$		
Number of Observation, n	5	5				
DF, (n-1)	4	4				
Mean, $\bar{F}$	5.60	5.65				
Variance, σ^2	0.236	0.113				
F-statistics	$\frac{0.236}{0.113} = 2.092$					
F-critical for $\alpha = 0.05$	6.388					

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Table 18: Fisher's F-Test for Split tensile Strength

S/N	F_1	F_2	$F_1 - \bar{F_1}$	$F_1 - \bar{F_2}$	$(F_1 - \bar{F_1})^2$	$(F_1 - \bar{F_2})^2$
C1	1.9	1.61	0.06	-0.15	0.004	0.024
C2	1.95	1.89	0.11	0.12	0.013	0.016
C3	1.88	1.94	0.04	0.17	0.002	0.029
C4	1.43	1.31	-0.41	-0.45	0.166	0.204
C5	2.03	2.08	0.19	0.31	0.037	0.097
				$\sum$	0.221	0.370
Number of Observation, n	5	5				
DF, (n-1)	4	4				
Mean, $\bar{F}$	1.84	1.77				
Variance, σ^2	0.055	0.0925				
F-statistics	$\frac{0.0925}{0.055} = 1.671$					
F-critical for $\alpha = 0.05$	6.388					

Since the F-statistics did not exceed the critical value from statistical table, the test confirmed that there is no significant difference between the predicted strengths (using Ibearugbulem's regression model) and the experimental results.

3. DISCUSSION AND CONCLUSION

The developed regression models (Equations 16-18) can effectively predict the 28-day compressive, flexural, and split tensile strengths of washed-gravel concrete at a 95% confidence level. These models were developed from experimental concrete strengths that are highly dependent on the constituent material properties, minimal human errors, and the laboratory conditions. In this study, the aggregates exhibited properties fully compliant with standard concrete production requirements. The washed gravel has a specific gravity of 2.7 and a water absorption of 0.8%, while the river sand shows values of 2.63 and 2.0%, respectively, all within the ASTM C128/C127 specified limits (<3% absorption). The aggregate impact value of 24.81% for the washed gravel confirmed its adequate toughness for structural applications, meeting BS 812-112 standards (<30%). Bulk density measurements of 1451.11 kg/m³ for the river sand and 1480.37 kg/m³ for the washed gravel met the ASTM C29/C29M requirements. Particle size distribution analysis (ASTM C136/C33) revealed both aggregates to be well-graded within specified finer and coarser limits, as shown in **Error! Reference source not found.** and **Error! Reference source not found.**, further validating their suitability for quality concrete production and supporting the reliability of the strength prediction models. Fisher's F-test confirms that potential errors (including human errors, laboratory conditions, and other sources) that may cause a difference between the laboratory results and the model predictions are within 5%.

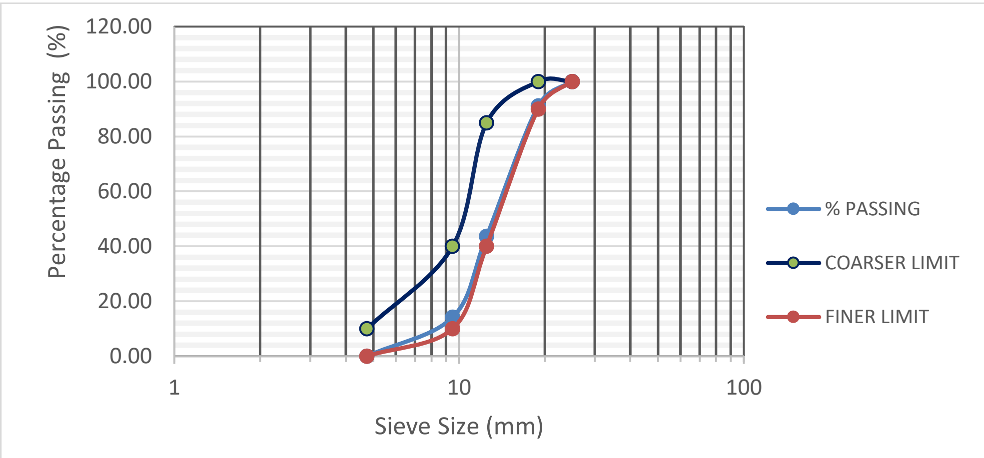

Fig. 1: Particle Size Distribution Curve for Coarse Aggregate (Washed Gravel)

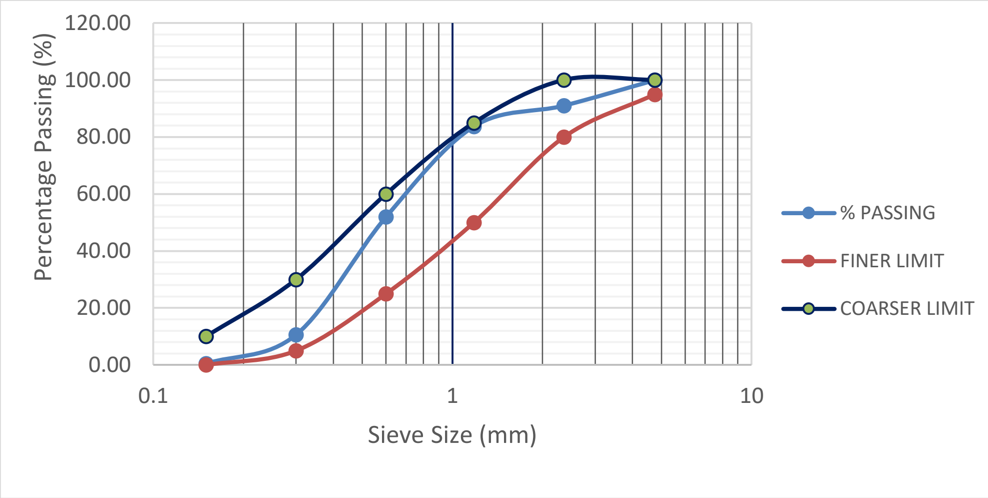

Fig. 2: Particle Size Distribution Curve for Fine Aggregate (River sand)

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